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The determination of the aerodynamic drag force on a parachute

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ABSTRACT: The use of the Buckingham Pi theorem in dimensional analysis suggests that the aerodynamic drag force that is exerted on parachutes is a quadratic function of speed, but it cannot determine the drag coefficients because they must be determined experimentally. In this article, experimental data are presented and used in two different, but complementary, ways to demonstrate that the drag force is quadratic, and to determine the corresponding drag coefficients. Curve-fitting a second-degree polynomial to the collected data makes it possible to verify directly that the nature of the force is quadratic and to estimate drag coefficients. Predicted terminal speeds of the parachutes are also used to determine the drag coefficients. A comparison of the two sets of results indicates that the combined use of the methods helps improve the accuracy of the drag force obtained.

INTRODUCTION

A parachute is an inflatable device made of soft fabric, such as nylon, which is used to slow down the speed of an object that moves through a fluid by creating a form of resistance commonly known as drag. Although parachutes are typically used to slow the descent of a person or object to the surface of Earth, or some other planet, from a high elevation, they are also utilised to increase the horizontal deceleration of a vehicle, such as an airplane or a space shuttle, after touchdown has been achieved.

Parachutes come in different shapes and designs, and represent various technologies, which are refined as new materials are produced and new techniques are invented. They are used in a wide variety of applications as well: military, personnel, cargo, and vehicle recovery, hobby, and education [1][2]. For example, each of the two Mars Explorer Rovers (MERs) launched by NASA in 2003 weighed 400 lbs and was equipped with a parachute of diameter 28 feet. They deployed successfully and slowed the descent of the MERs safely through the thin atmosphere of Mars [1].

The weights and sizes of parachutes vary greatly from the small sizes used by hobbyists to the large ones that are designed for the emergency braking of large military aircrafts. However, in all cases, it is necessary to estimate the time and distance necessary for the drag generated to slow the object to which the parachute is attached. The distance is necessary because engineers need to know whether or not, a fighter jet, for example, can land on a runway that is located, say, on the deck of an aircraft carrier. The time needs to be known because it is important to know for how long the pilots will be subjected to large deceleration forces.

The use of the Buckingham Pi theorem in dimensional analysis suggests that the aerodynamic drag force that is exerted on parachutes is a quadratic function of speed [2][3]. The purpose of this article is to use experimental data to demonstrate that such is the case [2]. The author does so in two different, but complementary, ways. First, curve-fitting a polynomial to the collected data is used. This allows one to verify directly whether or not the nature of the force is quadratic and to estimate the corresponding drag coefficients. Then, the terminal speed of a parachute obtained using an assumed quadratic force is used to determine the corresponding drag coefficient. Finally, the drag coefficients obtained from the two methods are computed.

MATHEMATICAL MODELS OF DRAG FORCES

In practice, it is often necessary to determine the coefficient of the drag of a parachute in order to assess its effectiveness in decelerating the movement of the payload being carried [1].

When a parachute of mass m is falling under the action of gravity, it is subjected to two distinct forces: its own weight, mg, and the drag force, F_D. Applying Newton's second law of motion in the vertical direction yields the following differential equation for the motion of the parachute:

$$m\frac{dV}{dt} + F_D - mg = 0, \tag{1}$$

where V is the vertical speed, t is the time, and g is the acceleration of gravity.

The mathematical nature of the drag force is difficult to determine a priori, except in special circumstances. Two of these are considered:

- Linear damping, as given by Stokes' law [3];
- Quadratic damping, as suggested by dimensional analysis [4].

Stokes Drag

For a sphere of radius a that is falling through a fluid of viscosity μ, after it has achieved a terminal speed V_t, the drag force F_D is given by Stokes' law [3]. This is:

$$F_D = 6\pi\mu V_t a\,. \tag{2}$$

It is often convenient to borrow the form of this drag force in order to obtain a first approximation to the drag force that is exerted on an object when the speeds are very small. In such cases, one rewrites Eq.(8) as:

$$F_D = k_1 V\,, \tag{3}$$

where k_1 is a coefficient to be obtained experimentally. Using Eq.(3) in Eq.(1), one gets:

$$m\frac{dV}{dt} + k_1 V - mg = 0\,. \tag{4}$$

For simplicity, it is assumed that the particle is released from rest. Thus, V(t = 0) = 0.

After separating the variables, and integrating and rearranging the terms, the solution of Eq.(4) is found to be:

$$\frac{V}{V_t} = 1 - e^{-\frac{k_1}{m}t} \tag{5}$$

Where V_t is the terminal velocity, which is achieved when the acceleration vanishes. It is given by:

$$V_t = \frac{mg}{k_1} \tag{6}$$

Whereas Stokes law assumes that terminal speed has been achieved, for particles that are dropped in a viscous medium, it takes a finite time interval and a finite displacement before they reach terminal speed. Thus, in laboratory experiments, for example, the application of Stokes' law first requires the determination of when and where terminal velocity is achievable. However, during the time preceding the attainment of terminal speed, the nature of the drag force is not known, a priori, so one needs to try other forms of damping.

Quadratic Damping

From dimensional analysis, it is determined that the drag force F_D on a smooth sphere of diameter d, moving through a viscous and incompressible fluid of mass density ρ and viscosity μ is given by:

$$F_D = \rho V^2 d^2 f\left(\frac{\rho V d}{\mu}\right)\,, \tag{7}$$

where f is some unknown function [4]. It is well-known that the form of this equation is generally valid for any object. In this case, it is written as:

$$F_D = k_2 V^2 \tag{8}$$

where k_2 is a coefficient that is determined experimentally. For an object that is submerged in an incompressible fluid in a region of space where free surface effects are negligible, dimensional analysis shows that the drag force has the mathematical expression given by:

$$F_D \equiv \frac{\rho A C_D}{2} V^2\,, \tag{9}$$

where ρ is the mass density of the fluid, A the cross-sectional area of the immersed object and C_D the so-called drag coefficient [4]. Although the latter is usually presumed to be constant, for simplicity, generally, such is not the case. Using Eq.(8) in Eq.(1) one gets:

$$m\frac{dV}{dt} + k_2 V^2 - mg = 0\,. \tag{10}$$

For simplicity, it is assumed that the particle is released from rest. Thus, V(t = 0) = 0.

After separating the variables, and integrating and rearranging the terms, the solution is found to be:

$$\frac{V(y)}{V_t} = \left[1 - e^{-\frac{2k_2}{m}y}\right]^{1/2}\,, \tag{11}$$

where y is the total distance travelled by the particle and V_t is the terminal velocity achieved when the acceleration vanishes. This is given by:

$$V_t \equiv \left(\frac{mg}{k_2}\right)^{1/2}; \tag{12}$$

General Drag

From the two preceding examples, one infers that, in general, the drag force can then be expressed as:

$$F_D = k_n V^n \tag{13}$$

Using Eq.(13) in Eq.(1), one gets:

$$m\frac{dV}{dt} + k_n V^n - mg = 0 \tag{14}$$

In this case, the terminal speed was shown to be given by the hypergeometric functions of Gauss [5-8].

EXPERIMENTAL DATA

It is considered that the motion of a sky diver, who has been released from a slow moving aircraft, is such that the diver falls straight down. The diver is equipped with a parachute that deploys at the appropriate elevation. While it is generally assumed to be a polynomial function of the speed, the aerodynamic force, F_D, on such a parachute is not known in general. It is estimated experimentally by testing falling objects like rockets that are equipped with similar parachutes; or by testing parachutes in wind tunnels or similar facilities [1]. The data so collected are plotted and, by curve-fitting different polynomials to the data, one can extract effective drag coefficients from them. One can also use the relationship between the terminal velocity and the drag force to explore the types of aerodynamic forces that are realistic and compatible with collected data.

Both methods are illustrated here using the data shown in Table 1, which were extracted from the experimental results of Nakka, who designed three small parachutes, tested each of them at different speeds, and measured the force applied to each parachute at the tested speeds [2]. The first two columns of Table 1 show his data [7]. When these data are plotted as shown in Figures 1 to 3, and the best parabola is fitted to each set, one gets the drag forces shown in column two of Table 2. However, when one uses the terminal speed as shown in Eq. (12), one gets the results shown in column three of Table 2.

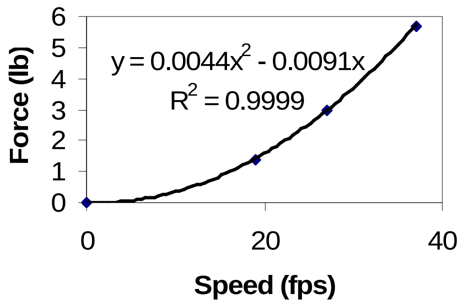

Figure 1: Drag force versus speed for a 25-inch parachute.

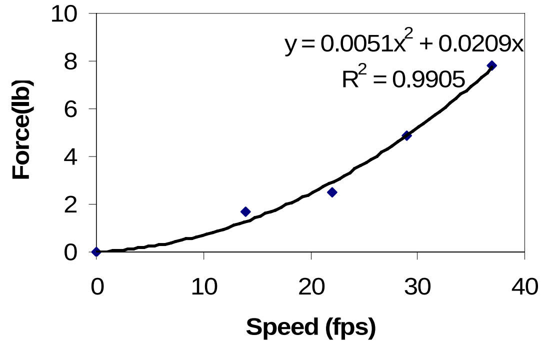

Figure 2: Drag force versus speed for a 30-inch parachute.

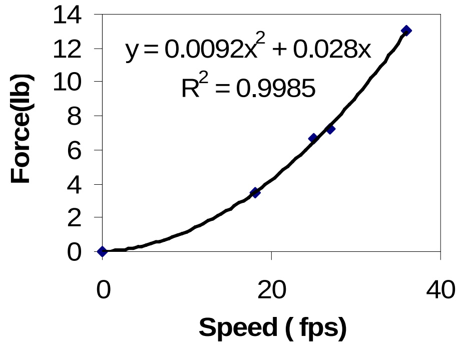

Figure 3: Drag force versus speed for a 1-metre parachute.

Table 1: Data estimated from the Figures of Richard Nakka [2].

Speed (fps)	Drag force(lb)	Drag Coefficient	$k_{2(\text{terminal speed})}$
	25-in Nylon parachute		$k_2 = 0.00431$
0	0		
19	1.4	0.55	
27	3.0	0.57	
37	5.7	0.64	
	30-in Nylon parachute		$k_2 = 0.00585$
0	0		
14	1.7	0.78	
22	2.5	0.62	
29	4.9	0.62	
37	7.8	0.63	
	1-m Nylon parachute		$k_2 = 0.01027$
0	0		
18	3.5	0.62	
25	6.7	0.58	
27	7.2	0.55	
36	13	0.59	

Table 2: A comparison of results curve-fit versus terminal speed.

Parachute	F_D from curve-fitting	F_D from terminal speed
25-inch	$0.0044V^2 - 0.0091V$, $R^2 = 0.999$ (From Figure 1)	$0.00431V^2$
30-inch	$0.0051V^2 + 0.0209V$, $R^2 = 0.991$ (From Figure 2)	$0.00585V^2$
1-metre	$0.0092V^2 + 0.0208V$, $R^2 = 0.999$ (From Figure 3)	$0.01027V^2$

PRACTICAL USE OF THE RESULTS

The results of Table 1 are detailed below.

A 25-inch parachute made of nylon can decelerate rockets weighing 2-4 lbs to touchdown speeds of 20 fps and 30 fps, respectively. Hence, a rocket of 3 lbs would have a touchdown speed of 26 fps. If we set $k_2 = 0.00431$ lb.s^2/ft^2 and weight = 3 lbs in Eq.(12), a speed of 26.38 fps is calculated. Nakka reports a speed of 26 fps [7]. Thus, the terminal speed calculated here and that obtained by Nakka only differ by 1.5% [2].

A 30-inch parachute made of nylon can decelerate rockets weighing 3-5 lbs to touchdown speeds of 20 fps and 30 fps, respectively. Hence, a rocket of 4 lbs would have a touchdown speed of 26 fps. If we set $k_2 = 0.00585$ lb.s^2/ft^2 and weight = 4lbs in Eq.(12), a speed of 26.14 fps is calculated. Nakka reports a speed of 26 fps [7]. Thus, the terminal speed calculated here and that obtained by Nakka only differ by 0.6% [2].

A 1-metre parachute made of nylon can decelerate rockets weighing 4-9 lbs to touchdown speeds of 20 fps and 30 fps, respectively. $k_2 = 0.010271$ lb.s^2/ft^2. Hence, a rocket of 6.5 lbs would have a touchdown speed of 25.15 fps. If we set $k_2 = 0.010271$ lb.s^2/ft^2 and weight = 6.5 lbs in Eq.(12), a speed of 25.15 fps is calculated. Nakka reports a speed of 25 fps [2]. Thus, the terminal speed calculated here and that obtained by Nakka only differ by 0.6% [2]. The corresponding drag coefficients obtained by using average speeds are summarised in Table 3.

Table 3: Sample parachute drag coefficients by Nakka.

Descent velocity (fps)	Descent mode	C_D
23 fps	Restrained	1.26
20 fps	Oscillating	1.60
16 fps	Gliding	2.40

THE DRAG COEFFICIENT OF A PARACHUTE

The drag coefficient of a parachute depends upon geometric factors and material properties, as well as the aerodynamic characteristics of its flight [6][7].

The geometric factors include the surface area of the canopy, the shape of the canopy, the length of the shroud lines and the aspect ratio (L/D) of the parachute (shroud line length)/(canopy diameter). The material properties include the porosity of the canopy and the permeability of the canopy. The aerodynamic characteristics of the parachute's flight include the speed of descent, the gliding characteristics and the patterns of air flow around the canopy.

A given parachute may descend in four different modes: restrained, gliding, oscillating (also called spiralling), or a combination of the last two. When the rates of descent are low, gliding tends to prevail; when they are intermediate, it is oscillations that prevail; and when the rates are high, mode combination is common. Thus, the drag coefficients can vary greatly, depending upon the mode of descent, even when the speeds are not very different, as shown in Table 3 [6][7].

While the shape of the canopy can vary considerably, from hemispherical, semi-ellipsoidal to parasheet, the drag coefficient is based not on the shape itself, but rather upon the area of the canopy. The drag coefficient increases with the length of the shroud lines. It also increases as the aspect ratio decreases. This may be due to the fact that small aspect ratios indicate a shape that is similar to that of a bluff body, whereas large aspect ratios simulate aerodynamic shapes [2-4].

The rate at which air escapes from the parachute through the surface area of the canopy is known as the permeability of the parachute. It is, of course, related to the porosity of the material of which the canopy is made. Experiments have shown that drag is not affected greatly by the permeability of the canopy.

It is a weak function of the speed of descent and, indeed, it decreases at higher velocities. The exact reasons for this behaviour are not clear at this time. However, it is speculated that this may be due to the increased porosity of the canopy that results from increased tension at high speeds, or it may be due to increases in the Reynolds numbers, as happens in flow past spheres and cylinders. Alternatively, it may be due to changes to the shape of the canopy that accompany high speeds; or even to a combination of these factors [6][7].

CONCLUSIONS

Two methods that are often necessary for the determination of drag coefficients of parachutes are presented. Curve-fitting a quadratic form to collected data was used because it allows one to verify whether or not the nature of the force is quadratic and to estimate the corresponding drag coefficients. When known, the terminal speed of the parachute can also be used to find the drag coefficients. It has been found that, while the first method allows one to verify the quadratic nature of the force, it introduces an error in the estimation of the drag coefficient. The second method assumes that the force is a quadratic function of the speed and, although it provides no inherent mechanism to verify that assumption, it gives a second estimate of the drag coefficient. A comparison of the results from these two methods indicates that they are very close and their combined use helps improve the accuracy of the determination of the drag force.

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